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Statistical Behavior of Multipair Crosstalk

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The literature of twisted-pair communication cables generally considers the probability distribution of both pair-to-pair crosstalk losses and their power sums to be normal on a decibel scale. A deficiency of this model is that the assumed normal distribution for pair-to-pair crosstalk loss must be truncated to correctly predict the power sum distribution. The appropriate truncation limit is selected to fit available cable data and varies with the data selected and the binder unit size. In this paper, we present experimental data from more than 600 cables, comprising 91,875 measurements, to show that the gamma distribution (with log variate) is a more satisfactory approximation to modeling the multipair crosstalk behavior. The predicted power sum distributions agree well with the data—without imposition of truncation. The gamma model leads to a more optimistic estimate of the number of digital systems which can be deployed in a cable. The result is, therefore, a lower cost per circuit, wider application of T-carrier system, or more efficient cable utilization than the normal model would permit.

I. INTRODUCTION

Baseband digital transmission systems,[1-10] such as T1, T1C, *SLC*™-40, *SLC*™-96 and other similar systems,[11-16] are being used increasingly on existing twisted multipair cables in short-haul trunk and subscriber loop networks as an economic alternative to analog voiceband transmission. This penetration will be further accelerated by the introductions of direct digital interfaces to ESS machines, digital central offices, and remote digital switching machines. New technologies, including large-scale integrated circuit and digital system approaches, and market growth in direct digital services to the customer premises[17-19] will also enhance the use of digital transmission. It therefore becomes important to understand the performance and the

limitations of various existing and future digital transmission systems on twisted-pair cables.

Crosstalk interference is a prime limitation on the transmission capacity and the repeater spacing of digital systems on twisted multipair cables. An important step in the design of digital systems and their associated engineering rules is the characterization of the power sums of pair-to-pair crosstalk losses.[20-31] The crosstalk power sum is the total crosstalk interference which appears on a given pair as a result of coupling from all disturbers on other pairs in the cable. Development of an accurate crosstalk model will allow maximum exploitation of the transmission capability of the vast existing network of twisted, multipair cables. This may be achieved by operating a larger number of digital systems in a given cable, by using a longer repeater spacing, or by using higher bit rate systems.

In the literature, the probability distributions of both pair-to-pair crosstalk loss and crosstalk power sum are often assumed to be normal on the decibel scale.* In this paper, we present a large amount of crosstalk data measured from 619 cables to show that the gamma distribution is a more satisfactory approximation than the conventionally accepted normal distribution for modeling multipair crosstalk behavior. A Computer-Operated Transmission Measurement Set[39,40] was used to carry out the large number of measurements.

II. CRITICAL REGION OF CROSSTALK DISTRIBUTION FOR ENGINEERING OF DIGITAL TRANSMISSION SYSTEMS

In engineering a digital transmission system in a twisted-pair cable, one is interested in the extreme tail region (e.g., 0.1 to 0.025 percent) of the probability distribution of crosstalk power sum. For example, the engineering rules of T-carrier system allows for a maximum of 50 repeater sections in metropolitan application and 200 repeater sections in outstate application. The engineering guidelines ensure that at least 95 percent of systems can withstand a minimum of 3 dB further degradation in signal-to-noise ratio before the system bit error rate exceeds 10^{-6} threshold. In other words, no more than 5 percent of systems can have less than 3 dB of margin in satisfactory signal-to-noise ratio. Notice that the 5-percent probability is the system objective and represents the sum of the probabilities of 50 metropolitan repeater sections or of 200 outstate repeater sections. This means, on each repeater section, the critical probability level is the 0.1-percent point for metropolitan T1 and the 0.025 percent point for outstate T1. Therefore, the practical value of a multipair crosstalk model should be judged by its accuracy in the extreme tail region below 0.1 percent point.

* I.e., lognormal on voltage or power scale.

III. MEASURED CROSSTALK STATISTICS

The crosstalk statistics of many different types of twisted multipair cables have often been characterized by measurements from only one sample cable. This section presents data measured from 545 cables to demonstrate that the measured crosstalk statistics varies substantially from cable to cable. A sample of one cable is too small to yield stable, representative results, especially in the critical extreme tail region. It is, therefore, important to employ data measured from a sufficiently large number of cables for accurate modeling of crosstalk behavior.

These 545 cables are 50-pair, 22-gauge, PIC* cables designed specifically with low capacitance (LOCAP) and improved crosstalk characteristics[32,33] for the T2 system operating at the 6.3 Mb/s rate. The within-unit far-end-crosstalk (FEXT) losses (pair-to-pair) and power sums were measured at 3.15 MHz at Baltimore. In such cables, each pair suffers from crosstalk interferences from 47† other pairs. A complete measurement of each cable yields 48 power sums due to 47 pair-to-pair, crosstalk losses. Thus one obtains a mean value, M_Q, and a standard deviation, σ_Q, of the 48 power sums for each measured cable. The value of M_Q and σ_Q varies substantially from cable to cable. The probability distributions of M_Q and σ_Q measured‡ from the 545 LOCAP cables are displayed on Figs. 1 and 2. It is seen that the standard deviation, σ_Q, varies from 0.95 to 4.9 dB and the mean value, M_Q, varies from 41.2 to 50 dB. Such large variations strongly underline the need for measurements of a large number of cables. Two different designs[47] of LOCAP cables were manufactured in 1972, which might have contributed to the large differences on Figs. 1 and 2.

The main reason for this instability is the small sample size (i.e., 48) of power sum from one cable. On the other hand, the sample size for pair-to-pair crosstalk from the 50-pair cable (48 potential digital pairs) is 1128.§ The large difference in sample sizes implies that the mean value and the standard deviation of pair-to-pair crosstalk loss are more stable than those of power sum. However, in application, the performance of the digital transmission system is controlled by the power sum distribution in the tail region (0.1 to 0.025 percent) of low crosstalk loss. Furthermore, the power sum distribution is strongly controlled by the few dominant disturbers with low pair-to-pair crosstalk loss. The measurements from one cable yield only a few "bad actors" and

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* Polyethylene-insulated conductor.

† Two pairs are used for order wire, fault location, alarms, and other maintenance purposes.

‡ Unfortunately, the pair-to-pair crosstalk statistics of the 545 LOCAP cables discussed in Section III were not documented. The author is unable to obtain the distribution of pair-to-pair crosstalk losses of these 545 cables measured prior to 1973.

§ I.e., $\binom{48}{2} = 48 \times 47/2$.

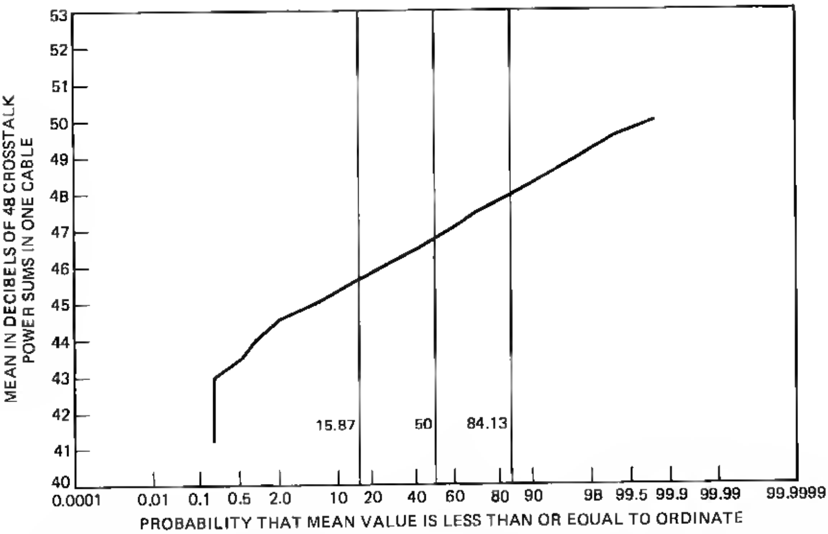

Fig. 1—Distribution of 545 mean values measured from 545 LOCAP, 50-pair, 22-gauge, PIC cables in Baltimore. Each mean value from each cable represents the average of 48 power sums of 47 pair-to-pair, EL-FEXT/1000 ft (in decibels) at 3.15 MHz.

hence provide very unstable statistics for power sum. The stability of pair-to-pair crosstalk loss in the middle range of the distribution is, unfortunately, outside of the range of interest in digital system engineering.

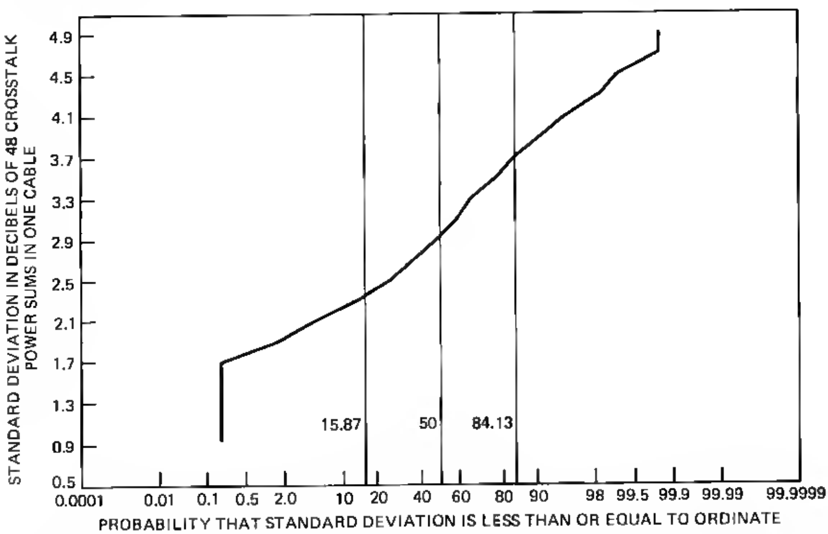

Fig. 2—Distribution of 545 standard deviations measured from 545 LOCAP, 50-pair, 22-gauge, PIC cables in Baltimore. Each standard deviation from each cable represents the standard deviation of 48 power sums of 47 pair-to-pair, EL-FEXT/1000 ft (in decibels) at 3.15 MHz.

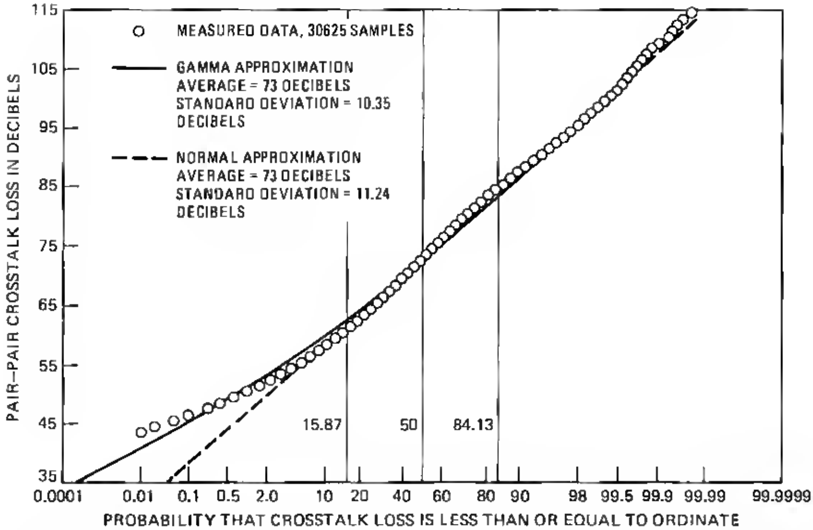

Fig. 3—Distribution of pair-to-pair, within unit, EL-FEXT/1000 ft at 3.15 MHz measured from 25 LOCAP, 50-pair, 22-gauge, PIC cables in Baltimore. The solid and dashed lines represent gamma- and normal-distribution approximations, respectively.

IV. MEASURED CROSSTALK DATA

This section presents several crosstalk data bases that will be employed for comparison of the normal and gamma models.

4.1 570 LOCAP cables measured at Baltimore

Data from 25 additional LOCAP cables also measured at Baltimore have been investigated.[42] The distribution of 30,625 samples of pair-to-pair FEXT at 3.15 MHz measured from these cables is shown on Fig. 3. The distribution of 27,360 samples of power sums of FEXT from the 570 cables is displayed on Fig. 7.

4.2 20 pulp cables measured at Kearny

Crosstalk data are also available for 20 cables measured at Kearny, New Jersey. They are 900-pair, 22-gauge, pulp cables composed of eighteen 50-pair binder units.[42, 46] Only 50 pairs in one binder unit from each cable are measured at 3.15 MHz. These FEXT data contain 24,500 samples of pair-to-pair crosstalk loss and 1000 samples of power sum. These data are shown in Figs. 4 and 8.

4.3 26 PIC cables measured at Baltimore

Twenty-six cables measured at Baltimore are 22-gauge, PIC cables consisting of twelve 50-pair cables, nine 100-pair cables, and five 200-

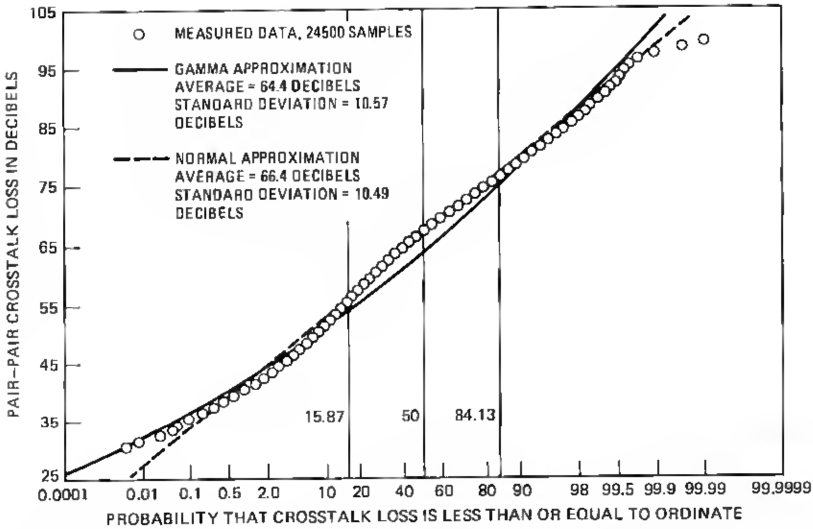

Fig. 4—Distribution of pair-to-pair, within unit, EL-FEXT/1000 ft at 3.15 MHz measured from 20 pulp, 22-gauge, 900-pair cables in Kearny, New Jersey. Each cable consists of eighteen 50-pair binder units. The solid and dashed lines represent gamma- and normal-distribution approximations, respectively.

pair cables.[42] Only 50 pairs in one group* from each cable are measured at 3.15 MHz. The FEXT data contain 31,850 samples of pair-to-pair crosstalk loss and 1300 samples of power sum. They are shown in Figs. 5 and 9.

4.4 *Four PIC cables measured at Atlanta*

The data described in subsections 4.1, 4.2, and 4.3 are all far-end-crosstalk (FEXT) data. It is also desirable to test the crosstalk model on near-end-crosstalk (NEXT) data. NEXT data from four 22-gauge, PIC cables measured at Atlanta, Georgia are examined[43] (three 100-pair cables and one 200-pair cable). Only 50 pairs in one group* from each cable were measured in this sample. The NEXT data at 0.772 MHz for pair-to-pair and power sum are shown in Figs. 6 and 10, respectively.

V. PROBLEMS OF THE NORMAL DISTRIBUTION MODEL

This section shows that the normal distribution model conventionally assumed is inadequate in that it requires the assumption of a truncation to the distribution for pair-to-pair crosstalk loss. The truncation parameter is empirically derived and must be defined on a case-by-case basis.

* The 50-pair group in the PIC cable is formed by combining either the adjacent 12-, 13-, and 25-pair units or the adjacent 25-pair units.

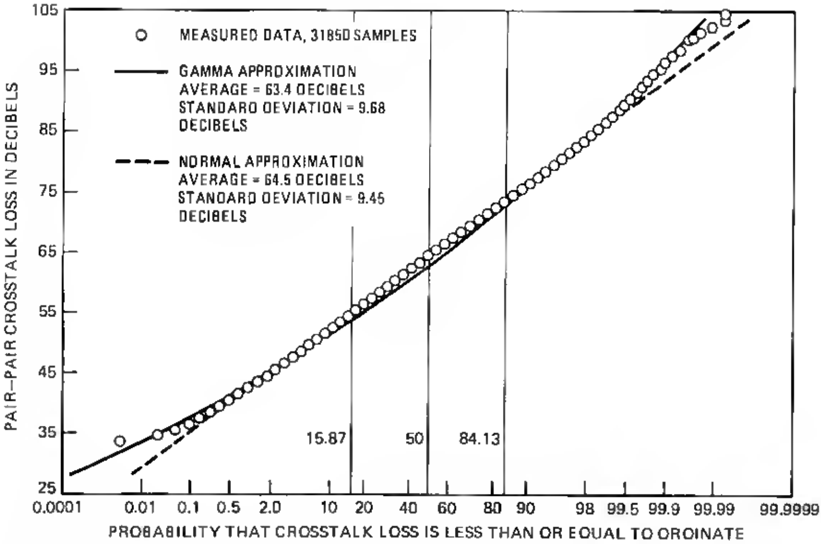

Fig. 5—Distribution of pair-to-pair, within unit, EL-FEXT/1000 ft at 3.15 MHz measured from 26 PIC, 22-gauge cables in Baltimore. The solid and dashed lines represent gamma- and normal-distribution approximations, respectively.

According to the analyses by Cravis and Crater[20] and by Matuda,[27] the pair-to-pair crosstalk loss for a given physical pair-to-pair separation is expected to be Rayleigh-distributed on the power scale. The rms value depends on the physical pair-to-pair separation and on the

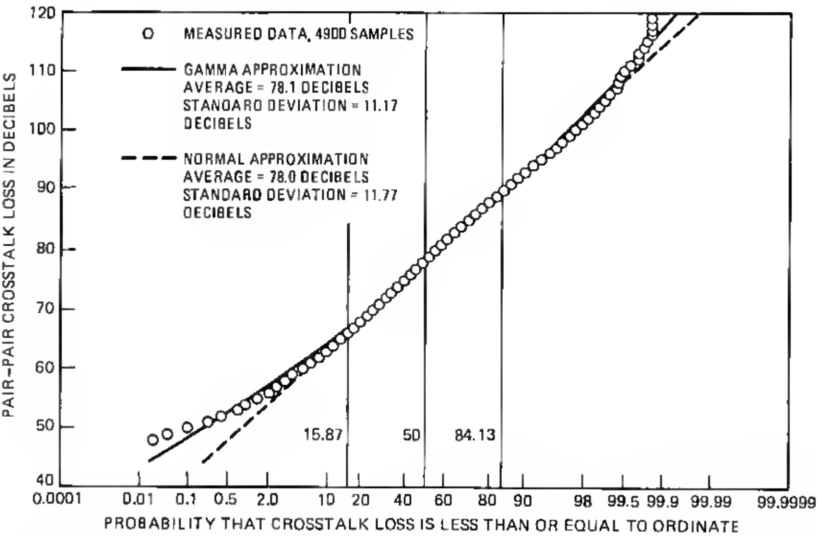

Fig. 6—Distribution of pair-to-pair, within unit NEXT at 0.772 MHz measured from 4 PIC, 22-gauge cables in Atlanta. The solid and dashed lines represent gamma- and normal-distribution approximations, respectively.

difference in twist lengths of the two crosstalking pairs. The ensemble of all the pair-to-pair crosstalk losses of a multipair binder unit, involving many different physical pair-to-pair separations and many different twist lengths, is a mixture of many Rayleigh distributions with different rms values. Such a mixture of many different distributions cannot be described exactly by a simple analytic function.

In the literature, both pair-to-pair crosstalk loss and power sum are often *assumed* to be normally distributed on a decibel scale. The mean and the standard deviation, and hence the entire distribution of the power sum, can then be calculated from those of pair-to-pair crosstalk losses through Wilkinson's approximation.[34]

However, many measured distributions of pair-to-pair crosstalk loss are not well fitted by normal distributions, particularly at the important low probability region. In the important tail region of low crosstalk loss, all the four sets of data in Figs. 3 to 6 consistently deviate from the normal distribution toward lower probability levels. In other words, the normal approximation is consistently too pessimistic in the critical region. Many other sets of data from smaller sample sizes also exhibit this behavior (e.g., Fig. 1 in Ref. 21).

The dashed lines on Figs. 7 to 10 show the power sum distributions predicted by the untruncated normal model. The predicted results quite obviously differ substantially from the measured data. Some further data in Ref. 21 indicate a similar problem.

The source of this gross error does not come from Wilkinson's

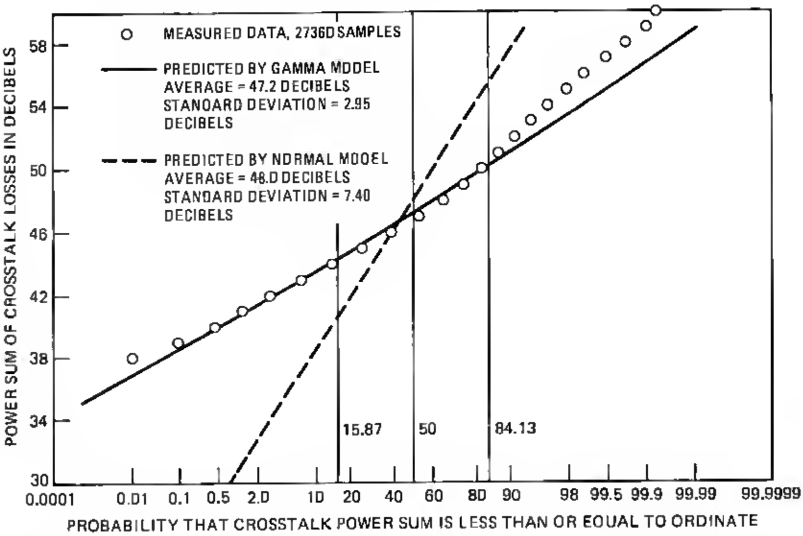

Fig. 7—Distribution of power sum of 47 pair-to-pair, within unit, EL-FEXT/1000 ft at 3.15 MHz measured from 570 LOCAP, 50-pair, 22-gauge, PIC cables in Baltimore. The solid and dashed lines represent predictions by gamma and untruncated normal models, respectively.

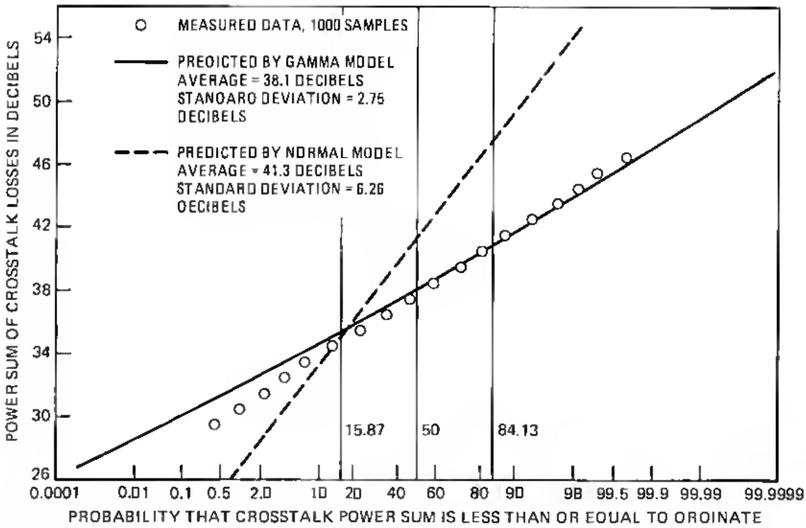

Fig. 8—Distribution of power sum of 49 pair-to-pair, within unit, EL-FEXT/1000 ft at 3.15 MHz measured from 20 pulp, 22-gauge, 900-pair cables in Kearny, New Jersey. The solid and dashed lines represent predictions by gamma and untruncated normal models, respectively.

approximation, but from the deviation of the pair-to-pair data from the untruncated normal distribution in the lower tail region (see Figs. 3 to 6). The power sum of many pair-to-pair crosstalk losses with a large standard deviation (e.g., $\sigma \simeq 10$ dB) is often dominated by only

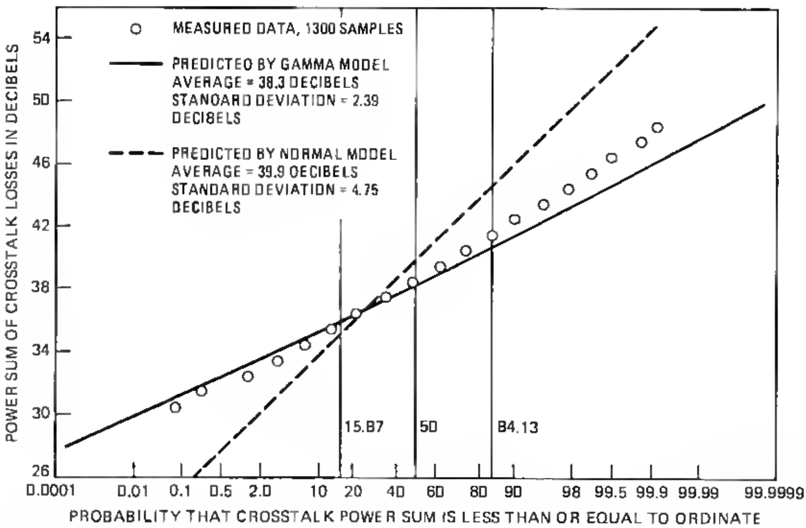

Fig. 9—Distribution of power sum of 49, within-unit, pair-to-pair, EL-FEXT/1000 ft at 3.15 MHz measured from 26 PIC, 22-gauge cables in Baltimore. The solid and dashed lines represent predictions by gamma and untruncated normal models, respectively.

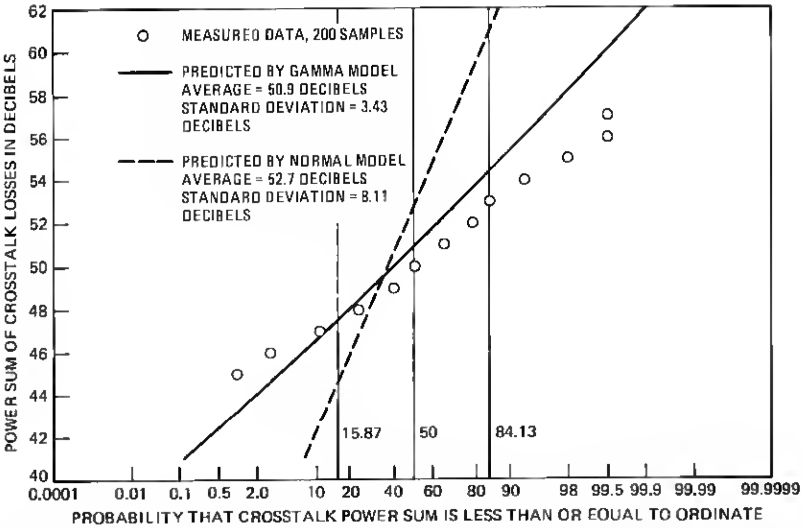

Fig. 10—Distribution of power sum of 49, within-unit, pair-to-pair, NEXT at 0.772 MHz measured from 4 PIC, 22-gauge cables in Atlanta. The solid and dashed lines represent predictions by gamma and untruncated normal models, respectively.

a few disturbers with low crosstalk loss. The power sum distribution is, therefore, strongly controlled by the behavior of the pair-to-pair distribution in the lower loss tail region where, unfortunately, the accuracy of the untruncated normal approximation is the poorest.

This problem has been encountered by many authors dealing with many different sets of crosstalk data. To remedy this deficiency, a truncated normal distribution is often used to approximate the pair-to-pair distribution.[21–24, 34, 37] In other words, the normal distribution for the pair-to-pair crosstalk losses is assumed to be truncated at $\pm c$ units of standard deviation from the mean value.

However, the available data indicate that the truncation point, c, of the pair-to-pair distribution must be carefully selected for each type of cable and each type of crosstalk mechanism to correctly predict the power sum distribution. No satisfactory single truncation point can be given for this model. For example, Fig. 11 indicates that the appropriate truncation point, c, is about 3.5 for FEXT at 3.15 MHz in 22-gauge PIC cables. On the other hand, Fig. 12 indicates that the appropriate truncation point should be 2.5 for 0.772 MHz NEXT in 22-gauge PIC cables. Furthermore, Fig. 13 shows that there is practically *no* suitable truncation point for 3.15 MHz FEXT in LOCAP cables. Hauschildt[21] has deduced an empirical formula from measured data for estimating the correct truncation point from the measured worst pair-to-pair crosstalk loss. Thus, the truncated normal distribution model is a *three*-parameter model with the mean, the standard deviation, and the truncation

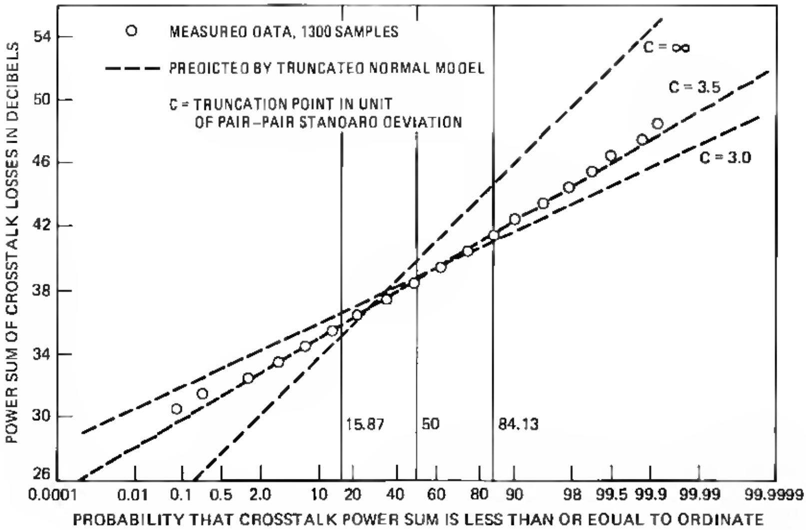

Fig. 11—Distribution of power sum of 49, within-unit, pair-to-pair, EL-FEXT/1000 ft at 3.15 MHz measured from 26 PIC, 22-gauge cables in Baltimore. The three dashed lines represent predictions by truncated normal model with truncation point $c = \infty$, 3.5, and 3.0.

point which must be characterized for each type of cable, each type of crosstalk mechanism, and each binder unit size.[24] Furthermore, there is no guarantee that it will work for all cables even with three adjustable parameters.

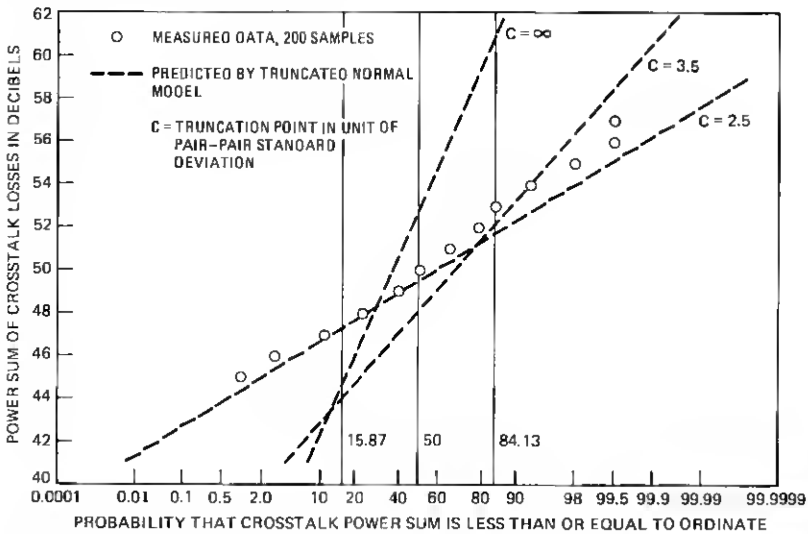

Fig. 12—Distribution of power sum of 49, within-unit, pair-to-pair, NEXT at 0.772 MHz measured from 4 PIC, 22-gauge cables in Atlanta. The three dashed lines represent predictions by truncated normal model with truncation point $c = \infty$, 3.5, and 2.5.

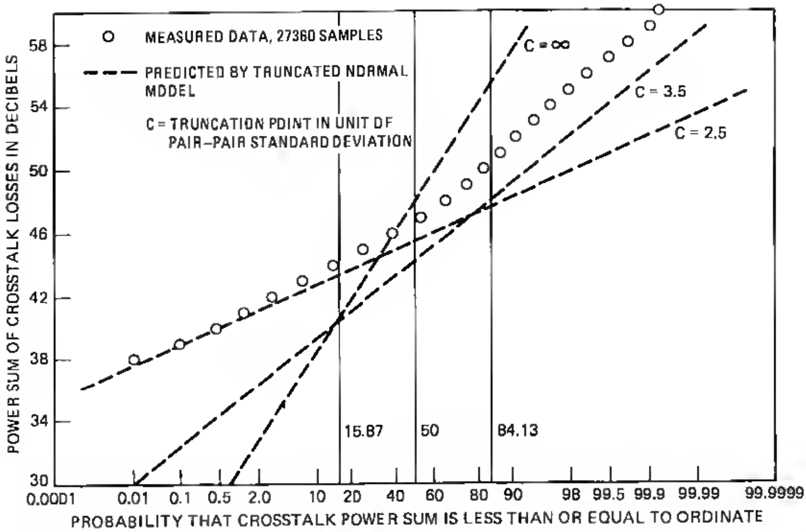

Fig. 13—Distribution of power sum of 47, within-unit, pair-to-pair, FEXT/1000 ft at 3.15 MHz measured from 570, LOCAP, 50-pair, 22-gauge, PIC cables in Baltimore. The three dashed lines represent predictions by truncated normal model with truncation point $c = \infty$, 3.5, and 2.5.

A finite sample measurement of a random variate (e.g., crosstalk loss) always yields a *finite* worst value even if the parent distribution of the variate is *untruncated*. The sample worst value varies randomly from one set of measurement (e.g., from one cable) to another. The probability distribution of the worst value (i.e., the extreme value) is the subject of extreme value statistics which have been studied extensively.[48–50] Therefore, the existence of a *finite* worst value from a *finite* sample measurement of cable crosstalk loss does not necessarily imply that the parent distribution of crosstalk loss is truncated.

In the next section, we show that the gamma distribution model with only two parameters provides more satisfactory results.

VI. THE GAMMA DISTRIBUTION MODEL

In this paper, we assume that both the pair-to-pair crosstalk losses and the power sums on the decibel scale are approximately gamma distributed. The details of the equations and the method for calculation of power sum distribution based on the gamma distribution model are given in Appendices A and B.

Figures 3 to 6 show the comparison of the pair-to-pair data with the normal and gamma approximations. The normal distribution fits the data closely in the middle range but deviates substantially from the data in both upper and lower tail regions. The gamma distribution

fits* the data reasonably well over the entire range. The ability of the gamma distribution to track the pair-to-pair data points closely in the lower tail region is crucial to its accuracy in the prediction of the power sum distribution.

Figures 7 to 10 show that the power sum distributions predicted by the gamma distribution model agree closely with the data. The gamma distribution is a two-parameter distribution. No truncation point is necessary in this model.

Among the four sets of data, the near-end-crosstalk (NEXT) data show the largest deviation from the gamma model because of the smallest sample size (only four cables). It appears that measurements from more than four cables are required for a critical comparison with the model.

From an application viewpoint, Figs. 7 to 10 indicate that the design of a digital system and its engineering rules based on the untruncated normal model is pessimistic. The use of the truncated normal model with $c = 3.5$ may be about right for some cases (e.g., Fig. 11) but may be still too pessimistic for other cases (e.g., Figs. 12 and 13). The gamma model is much closer to the measured data and provides a relatively more optimistic result.

VIII. GAMMA MOOEL VERSUS TRUNCATEO NORMAL MOOEL FOR DIGITAL SYSTEM DESIGN

Since the untruncated normal is obviously too pessimistic (see Figs. 7, 8, and 9), the truncated normal model with $c = 3.5$ is often used in the design of digital transmission system. Tables IV and V compare the predictions by gamma and truncated normal models at 0.1 and 0.025 percent points critical to the system design. It is seen that the truncated normal model with $c = 3.5$ is still too pessimistic by 1 to 7 dB, depending on cable and on application. A 7-dB difference in crosstalk power sum can be translated into a factor-of-5 difference in allowable maximum number of T-carrier systems in a given cable if the system is crosstalk-limited.

VIII. CONCLUSION

Far-end-crosstalk data have been obtained from measurements conducted on 570 LOCAP cables, 26 PIC cables, and 20 pulp cables. Near-end-crosstalk data have been obtained from four PIC cables. This large data base, consisting of 91,875 samples of pair-to-pair crosstalk losses

* The two parameters of the gamma distribution are determined by a least-squares fit of the gamma distribution on decibel scale to the data using Marquardt's algorithm (Refs. 35 and 36).

and 29,860 samples of crosstalk power sums, is better described by the gamma distribution than by the normal distribution.

At the critical probability level important to system design, the gamma model is more optimistic than the truncated normal model ($c = 3.5$) by 1 to 7 dB, depending on cable and application. A 7-dB difference in crosstalk power sum can mean a factor-of-5 difference in maximum allowable number of T-carrier systems in a cable. The resultant advantage of the gamma model is, therefore, lower cost per circuit, wider application of T-carrier system, or higher utilization of cable transmission network.

IX. ACKNOWLEDGMENT

I wish to express appreciation to F. C. Dunbar and E. Zarins for the far-end-crosstalk data and to T. F. McIntosh for the near-end-crosstalk data presented in this paper.

APPENDIX A

Basic Definitions in Gamma Distribution Model for Multipair Crosstalk Statistics

This appendix presents the basic definitions and equations used in gamma distribution modeling of the multipair crosstalk behavior.

Let

$$Q = -10 \log_{10} [10^{-x_1/10} + 10^{-x_2/10} + \cdots + 10^{-x_n/10}] \tag{1}$$

be the power sum of the n pair-to-pair crosstalk losses $[x_i]_{i=1}^n$ on the decibel scale. It is assumed that $[x_i]_{i=1}^n$ are independent and are identically distributed random variables. The distributions of both pair-to-pair crosstalk loss and the power sum on the decibel scale are assumed to be approximately gamma, i.e.,

$$P(x \le b) = \frac{\beta^\nu}{\Gamma(\nu)} \int_0^b x^{\nu-1} \cdot e^{-\beta x}\, dx \tag{2}$$

and

$$P(R \le a) = \frac{\alpha^\mu}{\Gamma(\mu)} \int_0^a R^{\mu-1} \cdot e^{-\alpha R}\, dR, \tag{3}$$

where

$$R = Q + 10 \log_{10} n, \qquad \text{dB} \tag{4}$$

and $\Gamma(\sim)$ denotes the gamma function. The range for x is from zero to infinity because the pair-to-pair crosstalk loss on the decibel scale is nonnegative.* On the other hand, the power sum of n pair-to-pair

* Strictly speaking, the normal distribution approximations for x and Q do not satisfy these conditions.

crosstalk losses on the decibel scale can be negative, and the lower limit is $-10 \log_{10} n$ dB. This is the basis for defining the new variable, R, by eq. (4) in order to use the standard definition of gamma distribution (3) for the power sums.*

Let us define

$$y_i = 10^{-x_i/10}, \qquad i = 1, 2, \cdots, n \tag{5}$$

as the pair-to-pair crosstalk losses on the power ratio scale, and

$$S = 10^{-Q/10} \tag{6}$$

as the crosstalk power sum on the power ratio scale. Based on these definitions, it can be shown that[38]

$$\nu = \left(\frac{M_x}{\sigma_x}\right)^2, \tag{7}$$

$$\beta = \frac{M_x}{\sigma_x^2}, \tag{8}$$

$$\mu = \left(\frac{M_Q + 10 \log_{10} n}{\sigma_Q}\right)^2, \tag{9}$$

$$\alpha = \frac{M_Q + 10 \log_{10} n}{\sigma_Q^2}, \tag{10}$$

$$\bar{y} = \frac{\beta^\nu}{(\beta + \lambda)^\nu}, \tag{11}$$

$$\sigma_y^2 = \frac{\beta^\nu}{(\beta + 2\lambda)^\nu} - \bar{y}^2, \tag{12}$$

$$\bar{S} = \frac{n \cdot \alpha^\mu}{(\alpha + \lambda)^\mu}, \tag{13}$$

and

$$\sigma_s^2 = \frac{n^2 \cdot \alpha^\mu}{(\alpha + 2\lambda)^\mu} - \bar{S}^2, \tag{14}$$

where

$$\lambda = (\ln 10)/10 \simeq 0.2303, \tag{15}$$

M_x and σ_x are the mean and the standard deviation, respectively, of x (on the decibel scale), M_Q and σ_Q are the mean and the standard deviation, respectively, of Q (on the decibel scale), $\bar{y}$ and σ_y are the mean and the standard deviation, respectively, of y (on the power ratio scale), and $\bar{S}$ and σ_s are the mean and the standard deviation, respectively, of S (on the power ratio scale). Substituting (5) and (6) into (1)

yields

$$S = \sum_{i=1}^{n} y_i, \tag{16}$$

which implies

$$\bar{S} = n \cdot \bar{y}, \tag{17}$$

and

$$\sigma_s^2 = n \cdot \sigma_y^2. \tag{18}$$

Equations (13) and (14) can be combined and rearranged to yield

$$\frac{\ln(\bar{S}) - \ln n}{\ln(\sigma_s^2 + \bar{S}^2) - 2 \ln n} = \frac{\ln(\alpha/(\alpha + \lambda))}{\ln(\alpha/(\alpha + 2\lambda))} \tag{19}$$

and

$$\mu = \frac{\ln(\bar{S}/n)}{\ln(\alpha/(\alpha + \lambda))}, \tag{20}$$

which are useful for power sum calculations as described in Appendix B. Equations (7), (8), (9), and (10) can also be inverted to yield relationships for calculations of mean and standard deviation from ν, β, μ, and α:

$$M_x = \frac{\nu}{\beta}, \tag{21}$$

$$\sigma_x = \sqrt{\frac{\nu}{\beta^2}}, \tag{22}$$

$$M_Q = \frac{\mu}{\alpha} - 10 \log_{10} n, \tag{23}$$

and

$$\sigma_Q = \sqrt{\frac{\mu}{\alpha^2}}. \tag{24}$$

APPENDIX B

Procedure of Power Sum Calculation Using Gamma Distribution Model

The procedure for the calculation of power sum distribution from the pair-to-pair crosstalk distribution can be broken down to the following six steps:

(*i*) Determine the two parameters ν and β of the pair-to-pair distribution by a least-squares fit* of the gamma distribution (2) to the data.

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* In this paper, we use Marquardt's algorithm (Refs. 35 and 36) to perform the nonlinear least-squares fitting on the decibel scale.

(*ii*) Substitute ν and β into equations (11) and (12) to obtain $\bar{y}$ and σ_y.

Table I—Parameters of crosstalk distribution: gamma model within unit FEXT/1000 ft 3.15 MHz LOCAP cables, Baltimore

Pair-to-pair 25 cables	ν	4.98
	β	0.681
	M_x(dB)	73.0
	σ_x(dB)	10.35
Power sum $n = 47$ 570 cables	μ	470
	α	7.35
	M_Q(dB)	42.2
	σ_Q(dB)	2.95

Table II—Parameters of crosstalk distributions: gamma model within unit FEXT/1000 ft 3.15 MHz

		26 PIC Cables Baltimore	20 Pulp Cables Kearny
Pair-to-pair	ν	42.9	37.1
	β	0.677	0.576
	M_x(dB)	63.4	64.4
	σ_x(dB)	9.68	10.57
Power sum $n = 49$	μ	533	400
	α	9.66	7.27
	M_Q(dB)	38.3	38.1
	σ_Q(dB)	2.39	2.75

Table III—Parameters of crosstalk distribution: gamma model near-end-crosstalk, 0.772 MHz, 4 cables, Atlanta

Pair-to-pair	ν	48.9
	β	0.626
	M_x(dB)	78.1
	σ_x(dB)	11.17
Power sum $n = 49$	μ	391
	α	5.76
	M_Q(dB)	50.9
	σ_Q(dB)	3.43

Table IV—Comparison of gamma and truncated normal model predicted power sum distribution at 0.025 percent point (outstate application with 200 repeater sections)

Gamma Model (dB)	Truncated Normal Model $c = 3.5$ (dB)	Difference (dB)	Figure Numbers
37.5	31.0	6.5	7 and 13
39.5	32.5	7.0	10 and 12
30.5	28.5	2.0	9 and 11

Table V—Comparison of gamma and truncated normal model predicted power sum distribution at 0.1 percent point (metropolitan application with 50 repeater sections)

Gamma Model (dB)	Truncated Normal Model $c = 3.5$ (dB)	Difference (dB)	Figure Numbers
38.5	32.5	6.0	7 and 13
40.8	35.5	5.3	10 and 12
31.2	30.0	1.2	9 and 11

(*iii*) Substitute $\bar{y}$ and σ_y into eqs. (17) and (18) to obtain $\bar{S}$ and σ_s.

(*iv*) Substitute $\bar{S}$ and σ_s into eq. (19) and solve* the nonlinear equation (19) for α.

(*v*) Substitute $\bar{S}$ and α into eq. (20) to calculate μ.

(*vi*) Substitute α and μ into eqs. (3) and (4) to obtain the predicted gamma distribution of power sum Q.

In some cases, the distribution of pair-to-pair crosstalk losses is not available. Only the values of M_x and σ_x are given. Under such conditions, we can use eqs. (7) and (8) to calculate ν and β from M_x and σ_x. Then we follow steps (*ii*) to (*vi*) to calculate the power sum distribution. The two power sum distributions calculated by the two different procedures would be identical if the pair-to-pair crosstalk loss were exactly gamma-distributed. In practice, some differences are expected because the pair-to-pair crosstalk data are not exactly gamma-distributed.

All the power sum distributions predicted by the gamma model on Figs. 7 to 10 are calculated by the procedure (*i*) to (*vi*). The parameters characterizing the 570 LOCAP cables, the 26 PIC cables, the 20 pulp cables, and the 4 PIC cables are summarized in Tables I to III.

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